Model- Based And Model - Free Schemes For Monitoring Mean And Variance Of Autocorrelated Process

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by

DR. ANNIE CHERIAN

ASSOCIATE PROFESSOR IN STATISTICS

BASELIUS COLLEGE, KOTTAYAM

Summary of the Project Report

In traditional application of control charts the observations from the manufacturing process are usually assumed to be independent and identically distributed. With development of advanced measurements and data collection technology, processes can be sampled at higher rates and the high frequency of sampling leads to data correlation. Also, in continuous flow processes like chemical processes, the data are correlated (English et. al. (1991)). Many authors have discussed the performance characteristics of standard control charts when applied to correlated observations. A basic conclusion that can be drawn from their studies is that correlation has a significant effect on the properties of the control charts that were investigated. When correlation is present in the data there are serious problems of not detecting the special causes that truly exist and giving false signals when there is no special cause.

For some processes, special causes can result in a simultaneous change in both the mean and the variance. In this case it is more reasonable to combine the mean and variance information on one scheme and look at their behavior jointly. Also, when the exact model of the process observations is not known then it is meaningful to consider model free schemes to monitor the process.

In this project existing control charts for simultaneous monitoring of the process mean and variance –both model based and model free – are studied and the performance of these charts in presence of correlation are evaluated. The study assumes an Autoregressive model of order one (AR(1)) for the process observations. Autocorrelation significantly affects both the process mean chart and the process variance chart. Positive autocorrelation in observations result in negative bias in traditional estimators of the standard deviation. Due to the under estimation of process standard deviation, the control limits for standard control charts for process mean become much narrow than desired. Narrow control limits, combined with autocorrelation in the observations plotted, could result in an average false alarm rate much higher than expected. That is, when autocorrelation increase the in-control ARL become much lesser than expected. But in the case of variance charts the in – control ARL increase when there is positive correlation and become less sensitive to increase in process variability.

The most commonly used method of SPC for autocorrelated data is the time-series modelling approach proposed by Alwan and Roberts(1988). The idea is to fit an appropriate time-series model for the process observations and obtain the one-step-ahead forecast. If the fitted model is exact, the forecast residuals are uncorrelated and follow $N(0, \sigma_\varepsilon^2)$ distribution so that any conventional chart can be employed to monitor the sequence of the residuals. Analysis of Run Length properties of residuals charts shows that these charts do not have the same run length properties as that of the charts applied to iid observations. The ARLs of the residuals chart , when there is a shift are significantly higher than those for the iid case. This increased out-of-control ARL is due to the fact that the forecast residuals recover quickly from the shift that occurred in the process mean level.

In the present study, the combination schemes applied to actual observations and residuals from a fitted time series model are considered. The performance of the combination schemes (1) EWMA chart of the observations and a Shewhart chart of residuals (2) EWMA of the residuals and EWMA of log of squared residuals and (3)the distribution free approach, Unweighted Batch Means (UBM) Chart applied to both original observations and forecast residuals are evaluated based on a simulation study. Computer programmes for computing the ARLs of the schemes under study are developed using R programming. Based on the study it can be concluded that EWMA of observations and Shewhart Individuals chart for residuals shows reasonable performance for all levels of autocorrelation. But the Unweighted Batch Mean chart applied to residuals is superior in performance to all the other schemes for detecting shifts in means. For detecting an increase in variance UBM chart meant for detecting change in the mean is equal in their performance to the other combined schemes.